

p-adic probability prediction of correlations between particles in the two-slit and neutron interferometry experiments

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¹

Abstract

We start from Feynman's idea to use negative probabilities to describe the two slit experiment and other quantum interference experiments. Formally by using negative probability distributions we can explain the results of the two slit experiment on the basis of the pure corpuscular picture of quantum mechanics. However, negative probabilities are absurd objects in the framework of the standard Kolmogorov theory of probability. We present a large class of non-Kolmogorovian probability models where negative probabilities are well defined on the frequency basis. These are models with probabilities which belong to the so-called field of p -adic numbers. However, these models are characterized by correlations between trials. Therefore, we predict correlations between particles in interference experiments. In fact, our predictions are similar to the predictions of the so-called nonergodic interpretation of quantum mechanics, which was proposed by V. Buonomano. We propose the concrete experiments (in particular, in the framework of the neutron interferometry) to verify our predictions on the correlations.

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1 Introduction

It is well known (see, for example, R.Feynman [1]) that probabilistic distributions which appear in the two slit experiment would not be described by the usual theory of probability (based on the axiomatic of A.N.Kolmogorov [2]). There were different attempts to propose new probabilistic theories to describe this situation (see, [3]-[6]). Despite of some of these theories were sufficiently fruitful from the formal mathematical point of view, they could not explain the physical matter of the two slit experiment, i.e., in particular, to answer to the question: *does a quantum particle go through one fixed slit or through both slits?*

In the present paper we also try to apply to the two slit experiment a new probabilistic theory. This is so called a p -adic theory of probability [7]. There probabilities might belong to a field of p -adic numbers \mathbf{Q}_p (as the field of real numbers \mathbf{R} , this field is a completion of the field of rational numbers \mathbf{Q} , see the next section for the main properties of p -adic numbers).² As probabilities do not belong to the segment $[0,1]$ of the real line, p -adic theory is a non-Kolmogorovean probabilistic model (see [9],[10] on the extensions of Kolmogorov's axiomatic). But our approach differs very much from previous non-Kolmogorovean descriptions of the two slit experiment [3]-[6]. Our theory is a frequency theory, i.e. it is formulated, not in the framework of a measure theory, but using a frequency definition of probability. Hence we have direct connection with physical reality (using relative frequencies) and may test in experiments some consequences of a p -adic probabilistic model for the two slit experiment.

The main prediction of our p -adic theory of probability is that the Kolmogorov (algorithmic) complexity of "random sequences" ω generated in the two slit experiment has the behaviour $K[(\omega)_n] = \log_p n$ where $(\omega)_n = (\omega_1, \dots, \omega_n)$ is the initial segment of ω of the length n . On the other hand, (at least formally) the Kolmogorov complexity of random sequences corresponding to independent trials has the behaviour $K[(\omega)_n] = n$.

Conclusion. *There exist correlations between quantum particles in the two-slit (and other interference) experiments.*

This prediction can be verified experimentally.

²First \mathbf{Q}_p -valued probabilities were used in so called p -adic(non-Archimedean) physics (see, for example, [8],[7] for these physical models).

As we hope that this paper should be interesting for experimentalists, we start directly from the experimental consequences of the p -adic probability predictions. In particular, we discuss the possible experiments to show the correlation between particles. We consider also the framework of the neutron interferometry [11] and propose some concrete experiments. In fact, these experiments are not so complicated (or expensive). They could be realized on the standard equipment of the neutron interferometry. If some of these experiments be successful, the pure particle picture of quantum mechanics should be justified and interference phenomenon should be regarded to the interaction between quantum particles and laboratory equipment.

The second part of the paper is devoted to the theoretical considerations. In particular, this part contains all primary facts about p -adic numbers, foundations of the frequency theory of probability (R.von Mises [12], 1919), non-Kolmogorovean model with p -adic probabilities and the Kolmogorov algorithmic complexity.

In the theoretical background our main idea is the following:

As geometry is not restricted to the Euclidean model, in the same way probability is not restricted to the Kolmogorovean model. As some physical phenomena cannot be described by the Euclidean geometry, in the same way some physical phenomena cannot be described by the Kolmogorov probability.

2 Experimental consequences for the two slit experiment

In the theoretical part of this paper we shall follow to the following chain of considerations:

experiment —> violations of the ordinary probabilistic properties —> negative probabilities solution —> p -adic frequency description of negative probabilities in the two slit experiment —> Kolmogorov complexity of p -adic collectives —> correlations between trials in the two slit experiment.

As a consequence, we have

Conclusion. Trials in the two slit experiment are not independent.

We have to test our prediction in physical experiments. At the moment, we do not know Where is an information about previous trials is accumulated? There are three (less or more natural) possibilities:

- (1) It is accumulated in the aperture. A new particle does not go through the aperture independently with previous particles.
- (2) Previous particles change a structure of the screen. The position of a new particle on the screen depends on these previous changes.
- (3) The source of particles accumulates an information about previous particles.

It seems to be that (1) and (3) are the most important possibilities.

What kind of experiments may test these hypothesis?

To exclude the correlations in the source of particle, we need a source of single particles which could not accumulate the information on previous particles. In the ideal case, we have to use a new source for a new particle.

To exclude the correlations due to (1) or (2), we have to change both shields (the shield with apertures and the screen) after each single particle. Hence, we should get only one point on every screen. Finally we should construct the histogram of points using a large statistical ensemble of screens with a single point on each of them.

We predict that *there should be no interference rings on this histogram or at least the interference should be very weak.*

Then we may realize experiments to separate hypothesis (1)-(3). For instance, we may change only screens after every experiment with a single particle.

These experiments seem to be very simple from the theoretical point of view. However, the discussion with scientists working in the quantum measurements showed that it should be technical problems to present a large ensemble of the *identical* equipment for the two-slit experiment.

Therefore, we have to propose more real experiments to verify our predictions. In the next section we shall discuss such experiments in the standard framework of the neutron interferometry. Then we shall go back to the two-slit framework and propose essentially new experiment to find the correlations between quantum particles.

3 Experimental consequences for the neutron interferometry

In fact, our predictions on the basis of the p -adic probability theory coincide with predictions of the so-called nonergodic interpretation of quantum mechanics, which was proposed by V. Buonomano [13].³ This interpretation uses the standard formalism of quantum theory, but it associates the expression $\langle A\psi, \psi \rangle$, which denotes the expectation value of the observable A of a system in state ψ , with the time average, rather than the ensemble average. The nonergodic interpretation of quantum mechanics was tested by few experiments in the framework of the neutron interferometry [14] - [16]. However, the results of these experiments seem to against the nonergodic interpretation of quantum mechanics. Therefore, they are more or less against our p -adic probabilistic predictions.

In the framework of the neutron interferometry we get the same predictions as for the two slit experiment: *all the interference in interferometers is a result of the dependence (correlations) of the detection events*. Therefore, I propose to reduce these correlations which should result in reduced visibility of the interference fringes. In the ideal case, if there are no correlations at all, one should see no interference.

In contrary all people working on neutron interferometry believe that detection events are Poissonian distributed and therefore completely independent from each other. For example, M. Zawisky [16] has done some experiments which tested the distribution of the output beams. It was found no significant deviation from the poissonian statistics within an accuracy of approximation 2%. Nevertheless a visibility of 50% of interference fringes in the outgoing beam was detected.

However, although there were no evidence for deviations from Poissonian statistics, it would be interesting to repeat the statistical analysis with more accuracy. To do this, we need to know how much difference we could expect between p -adic and Poissonian statistics. Of course, if the effect is too small, then it probably will be difficult to measure it with neutron interferometry.

At the moment, we cannot estimate accuracy because the p -adic probabilistic distribution is a type of hidden variables distribution. Our analysis

³I should like to thank H. Rauch and J. Summhammer who had pointed me to this connection.

implies only that "this is a p -adic distribution", but we cannot describe the concrete form of this distribution. For example, is it p -adic uniform distribution or not? Moreover, there is the parameter of the model: a prime number p . There can be 2-adic, 3-adic, 1997-adic distributions. Therefore it is not easy to get some predictions on the accuracy.

The possibility to prove our theory would be to increase the independence of all detection events and to measure the reduced visibility as a function of this independency. The general proposal is to change the source, interferometer and phase shifter after each single event. But it seems to be impossible in practice.

M. Zawisky proposed (in a private discussion) a much cheaper and easy way to perform experiment: If there are some memory effects in the system (neutron-interferometer, phase-shifter, source), then these effects must be time independent, otherwise the visibility would increase after each experiment. If we put different beam attenuators in front of the interferometer and if we measure the visibility with different input intensivities (but with same particles numbers therefore the measurement time depends on the beam attenuation) one should see such time dependent memory effects.

The most realistic is another experiment: to disturb the interferometer at the beginning of each measurement cycle to guarantee zero visibility at the starting point. In this experiment we may hope that we shall get non-Poissonian distribution.

Another idea is to destroy the time dependence by using the time factor: to repeat experiment after sufficiently long time. Of course, the main question is : What is a meaning of "long time"? From the p -adic point of view we have the following exponential scale: $t = p^n$, i.e. $t_0 = 0$ (first experiment), $t_1 = p$ (second),..., $t = p^n((n - 1)\text{-experiment})$,... Of course, there is still a problem of the parameter p . The only possibility is to start with $p = 2$. If the Poissonian structure is not destroyed (i.e. time-scale is too small), then to try $p = 3$, and so on. If the interference picture is not totally disappeared, we could hope that at least it will become more and more unsharp with increasing of p .

4 Random two-slit experiment

It seems to be that the following experiment should delete or at least make weaker the memory effects in the shield with slits.

Let us consider the shield with a large number of slits: $S = \{s_1, \dots, s_N\}$, $N = 2^k$. There is a device \mathcal{D} which can open and close slits. Consider two generators of (ordinary) pseudo-random numbers ξ and η . For example, these numbers can be chosen uniformly distributed on the set $\{1, \dots, N\}$. According to the values of these random generators \mathcal{D} open only two slits, s_ξ, s_η , for each particle generated by the source of single particles \mathcal{I} . Therefore, for any quantum particle registered on the screen, we are in the framework of the two slit experiment (of course, if $\xi = \eta$ for some trial, then we have one slit experiment which is considered as a particular case of the two-slit experiment). However, we need a separate screen for each particle registration.

Let us fix some configuration of the two-slits, $\alpha = (s_{i_1}, s_{i_2})$. After a large number of experiments we collect all screens corresponding to the experiments with $\xi = i_1$ and $\eta = i_2$ and construct the corresponding distribution of points by the projection to the unique screen.

From the point of view of the ordinary quantum mechanics we should get the standard interference picture which corresponds to the "pure two-slit experiment" with the slits $\alpha = (s_{i_1}, s_{i_2})$.

However, our p -adic theory predicts that in the ideal case the interference picture should disappear. The ideal case means that $N \rightarrow \infty$. In any case we predict that the interference picture should become weaker and weaker with the increasing of N .

5 Frequency theories of probability

The p -adic frequency probability theory is a natural extension of the von Mises theory [12] where probabilities were defined via a principle of statistical stabilization of relative frequencies. According to this principle, the statistical sample

$$x = (x_1, x_2, \dots, x_n, \dots), x_j = 0, 1, \quad (1)$$

is said to be a *collective* if there exist limits of relative frequencies $\nu_N(0) = n(0)/N$ and $\nu_N(1) = n(1)/N$ where $n(\alpha)$, $\alpha = 0, 1$, are numbers of realizations

of the labels α in the first N trials.⁴ The limits of these frequencies are probabilities in the framework of von Mises frequency probability theory.

The main advantage of the Mises approach with respect to the Kolmogorov one is that in the first one there is some kind of an underground level before probabilities. This is the level of collectives (random sequences). There are some situations in physics (in particular, the two slit experiment) where we might not compute all probabilities (at least, exactly), but we might extract some properties of the corresponding random sequences using the known probabilities.

The main line (a curve?) of our ideas is the following one.

The first thing which we know on the basis of the two slit experiments is that the corresponding random sequences are not Mises' collectives (because probabilities have unusual properties). May we generalize the Mises notion of the collective to get random sequences which are more adequate to the two slit experiment? Yes, we can do this. The most general extension of Mises' theory is provided by the following scheme [7]:

Let τ be an arbitrary topology on \mathbf{Q} . The statistical sample (refe1) is said to be a τ -*collective* if limits of relative frequencies $\nu_N(\alpha)$, $\alpha = 0, 1$, exist with respect to the topology τ . These limits belong to a completion \mathbf{Q}_τ of \mathbf{Q} . The topology τ is said to be a topology of statistical stabilization. For example, if τ is corresponding to a metric ρ on \mathbf{Q} , then the τ -stabilization means that $\rho(\nu_N(\alpha), \nu_M(\alpha)) \rightarrow 0$, $N, M \rightarrow \infty$, $\alpha = 0, 1$. In particular, if $\rho_R(x, y) = |x - y|_R$ is the ordinary real metric, we obtain the old Mises theory.

The topology of statistical stabilization τ is a parameter of a physical model. There are many physical experiments where τ is the ordinary real topology. These experiments generate Mises collectives. However, there are some experiments where topologies of statistical stabilization are more exotic. We think that the two slit experiment and other quantum interference experiments belong to the last class of experiments.

The main (and very hard) problem is to find the right topology τ of statistical stabilization corresponding to the fixed physical experiment \mathcal{E} . In many cases (especially in quantum experiments) this is really impossible to find τ exactly. However, sometimes the known information about properties

⁴Of course, in applications this means stabilization of digits in the decimal expansion of relative frequencies

of statistical samples generated by \mathcal{E} gives us the possibility to describe a class $C(\mathcal{E})$ of possible topologies of statistical stabilization. Then we can study theoretically the properties of τ -collectives, $\tau \in C(\mathcal{E})$, and obtain new properties which we cannot see directly from a statistical data for \mathcal{E} . Using these new (theoretical) properties, we may try to explain some problems connected with \mathcal{E} . Of course, it would be only a theoretical explanation To confirm it, we need to propose new physical experiments. This was the main line of our ideas.

This scheme is realized in the present paper for the two slit experiment. We shall show that if $\mathcal{E} = \mathcal{E}_2$ is the two slit experiment, the class $C_p(\mathcal{E}_2)$ of p -adic topologies on \mathbf{Q} seems to be adequate to this experiment. Then we shall study the properties of generators of random numbers for p -adic topologies. Using these properties, we could get some new predictions.

We notice that our p -adic topologies of statistical stabilization are not so exotic. According to the famous theorem of numbers theory (Ostrovsky theorem [17]), every metric on \mathbf{Q} of the form $\rho(x, y) = |x - y|$ where $|\cdot|$ is an absolute value (valuation) on \mathbf{Q} coincides with the real one or with one of p -adic metrics.

6 p-adic frequency realization of negative probabilities

The field of real numbers \mathbf{R} is constructed as the completion of the field of rational numbers \mathbf{Q} with respect to the metric $\rho(x, y) = |x - y|$, where $|\cdot|$ is the usual absolute value . The fields of p -adic numbers \mathbf{Q}_p are constructed in a corresponding way , by using other absolute values. For any prime number the p -adic absolute value $|\cdot|_p$ is defined in the following way. At the first , we define it for natural numbers . Every natural number n can be represented as the product of prime numbers : $n = 2^{r_2}3^{r_3}\cdots p^{r_p}\cdots$. Then we define $|n|_p = p^{-r_p}$, we set $|0|_p = 0$ and $|-n|_p = |n|_p$. We extend the definition of p -adic absolute value $|\cdot|_p$ to all rational numbers by setting for $m \neq 0$: $|n/m|_p = |n|_p/|m|_p$. The completion of \mathbf{Q} with respect to the metric $\rho_p(x, y) = |x - y|_p$ is a locally compact field \mathbf{Q}_p . It is well known , see [17] , that $|\cdot|$ and $|\cdot|_p$ are the only possible absolute values on \mathbf{Q} . The p -adic absolute value satisfies the strong triangle inequality : $|x + y|_p \leq \max(|x|_p, |y|_p)$. For

any $x \in Q_p$ we have a unique canonical expansion (converging in the $|\cdot|_p$ -norm) of the form

$$x = a_{-n}/p^n + \cdots a_0 + \cdots + a_k p^k + \cdots = \dots a_k \dots a_0, a_{-1} \dots a_{-n}, \quad (2)$$

where $a_j = 0, 1, \dots, p-1$, are the "digits" of the p -adic expansion.

Now we fix the prime number p and choose the p -adic topology as the topology of statistical stabilization , i.e. consider p -adic collectives as random sequences. The following mathematical result [7] is very important for our further considerations :

Every p -adic number x might be realized as a p -adic frequency probability.

For example,every rational number may be realized as a p -adic probability. There are such "pathological" probabilities (from the point of view of the usual theory of probability) as $\mathbf{P}(A) = 2$, $\mathbf{P}(A) = 100$, $\mathbf{P}(A) = 5/3$, $\mathbf{P}(A) = -1$; it may be possible that $\mathbf{P}(A) = i_p = \sqrt{-1}$,if $p = 1(mod4)$, because in this case i_p exists directly in Q_p .

The possibility to get negative probabilities using the frequency definition is the most important motivation to choose the p -adic topologies as topologies of statistical stabilization for the two slit experiment, $C(\mathcal{E}_2) = C_p(\mathcal{E}_2)$.

We continue the chain of our considerations. As we know from papers of R.Feynman [3], all problems of a probabilistic description of the two slit experiment might be solved on the basis of negative probability distributions.⁵ As these probabilities may be realized as p -adic frequency probabilities, we may assume that the two slit experiment generates p -adic collectives (random sequences with statistical stabilization in one of p -adic topologies).

Remark. We are not sure that we have found the exact class of topologies of statistical stabilization for the two slit experiment. Probably C_p is only the class of toy topologies which present only one specific property π_{ran} of random sequences generated by two slit experiment. In the mathematical description this property is realized as negative probabilities. The p -adic topologies present only this particular property of random sequences in the two slit experiment. We wish to find a physical counterpart π_{phys} in the two slit experiment corresponding to the property π_{ran} .

⁵All Feynman's investigations were heuristic, because negative probabilities were meaningless from the mathematical point of view.

7 Algorithmic complexity of random sequences generated by the two slit experiment

The property π_{ran} can be described on the basis of Kolmogorov's ideas about the algorithmic complexity of random sequences [18]. Kolmogorov's idea was to define random sequences on the basis of a notion of a complexity of their finite segments. As usual, finite vectors $x = (x_1, \dots, x_n)$, $x_j = 0, 1$, are called words with respect to the alphabet $\{0, 1\}$.

Definition. (A. N. Kolmogorov) Let \mathcal{A} be an arbitrary algorithm. A complexity of a word x with respect to \mathcal{A} is

$$K_{\mathcal{A}}(x) = \min l(\pi),$$

where $\{\pi\}$ are the programs which are able to realize the word x with the aid of \mathcal{A} .

This definition depends very much on a structure of \mathcal{A} . But A.N.Kolmogorov proved the following theorem, which was a good justification of this definition.

Theorem. There exists such algorithm \mathcal{A}_0 (optimal algorithm) that

$$K_{\mathcal{A}_0}(x) \leq K_{\mathcal{A}}(x) \quad (3)$$

for every algorithm \mathcal{A} . As usual, (refe3) means that there exists such constant C that

$$K_{\mathcal{A}_0}(x) \leq K_{\mathcal{A}}(x) + C$$

for all words x . An optimal algorithm \mathcal{A}_0 is not unique.

Definition .The complexity $K(x)$ of the word x is equal to the complexity $K_{\mathcal{A}_0}$ with respect to one fixed (for all considerations) optimal algorithm \mathcal{A}_0 .

A.N.Kolmogorov proposed to use the notion of the complexity of a finite word to try to define a random sequences with the aid of complexities of their finite segments. The idea of Kolmogorov was very natural. He proposed to consider a sequence $\omega \in \Omega$ as a random sequence, if finite segments $(\omega)_n = (\omega_1, \dots, \omega_n)$ of this sequence had complexities which are approximately equal to n . Thus,a sequence ω is a random sequence in the Kolmogorov sense iff it is impossible to find programs π_n ,generating words $(\omega)_n$, with lengths

$l(\pi_n) \ll n$. We need a word with a length not less than the length of the segment of ω for coding this segment.⁶

In [7] we estimated the Kolmogorov complexity of p -adic collectives. The main result is that complexity of the initial segments of a p -adic collective has the asymptotic $\log_p n$. Hence the Kolmogorov complexity for p -adic collectives is essentially less than the same complexity for ordinary random sequences (Mises' collectives). Therefore previous trials contain some information about the next trial. In particular, these trials are not independent. On the other hand, correlations between trials are sufficiently weak since the complexity $K((\omega)_n)$ increases as $\log_p n$. Thus we are not able to predict a lot about the next trial on the basis of the previous results.

Concluding remarks. 1). We wish again to notice that we are not sure that statistical samples of the two slit experiment are p -adic collectives. We cannot test this hypothesis directly (because we have not statistical data for separate slits). Probably the corresponding topology of statistical stabilization is much more complicated. We only extract one property of this topology which has the adequate p -adic description. As a consequence we obtain correlations between trials.

2) We note that log-complexity is sufficiently high complexity. Therefore, it is natural that it could be identified with the linear complexity (which corresponds to ordinary random sequences) in some experiments. In fact, the experiments have to distinguish two following hypothesis: $H_{Kol} = \{ \text{random sequences generated in interference experiments have the linear asymptotic of complexity} \}$ and $H_p = \{ \text{random sequences generated in interference experiments have the logarithmic asymptotic of complexity} \}$. As we have already told, we could not be sure that the log-behaviour is the right behaviour for interference experiments. Probably, $K((\omega)_n) = f(n)$, where $f(n)$ is some other function which increases slower than the linear function. Moreover, it should be that different interference experiments have different asymptotic behaviour, i.e. $f(n) = f_{\mathcal{E}}(n)$. In particular, it should be that $f(n) = \log_{p(\mathcal{E})} n$. It should be possible to classify interference phenomenon on this basis (2-adic interference and 1997-adic interference).

3). The experiments to verify log-correlations could be realized on the basis of the existed equipment for the neutron interferometry. The main

⁶In fact, the situation is not so simple from the mathematical point of view. We need to use more complicated notions of complexity.

problem is to attract physicists working in the neutron interferometry to realize these experiments. It is not easy, because there is the general opinion that the former experiments proved the independence of trials in the interference experiments.

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